Some Studies on Different Power Allocation Schemes of Superposition Modulation

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Abstract—**Superposition Modulation/Mapping (SM) is a newly evolving modulation technique in which the conversion from binary digits to symbols is done by linear superposition of the binary digits instead of bijective (one-to-one) mapping. Due to linear superposition, the symbol distribution of the data symbols thus formed are Gaussian shaped which is capacity achieving without active signal shaping. In this paper, a detailed study on SM has been presented with respect to its different power allocation schemes namely Equal Power Allocation (EPA), Unequal Power Allocation (UPA) and Grouped Power Allocation (GPA). Also, it has been shown that SM is more capacity achieving than the conventional modulation technique such as Quadrature Amplitude Modulation (QAM).**

Index Terms—**EPA, UPA, GPA, PSM, IDM, BICM, LDHC**

I. Introduction

Shannon's determination of the capacity of linear Gaussian channel is a landmark contribution in the history of communications. According to his theory, the capacity of a Gaussian channel corresponds to the maximum mutual information between channel inputs and outputs [1, 2]. Thus a Gaussian channel capacity is achieved if and only if the channel outputs have Gaussian-like distribution. This is possible only if the channel inputs are nearly or at least Gaussian. Active signal shaping techniques such as trellis shaping [3] and shell mapping [4, 5] have been proposed by many to produce Gaussian-like symbol distribution. The ultimate gain obtained due to signal shaping is almost 1.53 dB.

A multi-level coded transmission scheme has been designed in [6], where it has been shown that large signal constellation and active signal shaping are not mandatory for approaching the Gaussian channel capacity. In this scheme, each encoder output has been mapped into an independent signal constellation. Thus regardless of the Signal-to-Noise Ratio (SNR), active signal shaping has not been used in this scheme.

In the last many years, several researchers have proposed various schemes of transmission which employs linear superposition [7–9] to produce Gaussian-shaped symbols. The basic feature behind these techniques is the weighted superposition of the binary antipodal symbols.

Digital modulation techniques are generally bijective in nature and there is a one-to-one mapping of binary bits to data symbols. However, achieving both power and bandwidth efficiency simultaneously is a difficult task with conventional methods. A modulation technique is capacity achieving if it is both power and bandwidth efficient. Thus a non-bijective modulation scheme called SM has been proposed in [10] which has the capacity of achieving both power efficiency and bandwidth efficiency simultaneously through proper power allocation. It is a modulation technique in which the conversion from binary bits to data symbols is done by a certain form of linear superposition instead of bijective (one–to–one) mapping. The data symbols thus obtained after mapping the bits are often Gaussian–like and hence communication systems employing SM has a good theoretical potential to approach the capacity of Gaussian channels without using active signal shaping.

A comprehensive study of Phase Shifted Superposition mapping (PSM) has been presented in [11] with particular focus on iterative demapping and decoding. The symbol distribution of PSM is geometrically Gaussian-like and thus PSM has a good potential to approach the capacity of Gaussian channels.

At high SNRs, BICM [12] has been recognized as an excellent technique to approach the channel capacity also called the 'Shannon limit'. Typically, high-order QAM is used to achieve a high bandwidth efficiency, with an interleaver placed between the encoder and the signal mapper to exploit the bit diversity of QAM. With the same structure and replacing QAM with Superposition Mapping (SM), we obtain a transmission technique which forms a well-known structure called Bit-Interleaved Coded Modulation with Superposition Mapping (BICM-SM) [13].

Repetition codes are necessary to separate the superimposed chips. Moreover, parity check codes are required to combat the effects of white noise. A novel concept of coding which is a serial concatenation of low-density parity-check (LDPC) code [14] and repetition code has been proposed for SM systems with arbitrary power allocation. This coding approach called Low Density Hybrid Check (LDHC) coding fulfills both the above requirements and hence is the main coding technique proposed for Superposition Modulation [15].

The remainder of this paper is organized in the following manner. Section II gives a brief introduction on the Gaussian channel model. Section III presents the basic concepts of Superposition Mapping along with a thorough description of the different power allocation techniques. Section IV presents the simulation results. Finally, Section V summarizes the paper and outlines some open topics for future work.

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II. Gaussian Channel Model

The most popular channel model for modern digital communication systems is the discrete-time additive white Gaussian noise (AWGN) channel model. It can model the fundamental effects of communication in a noisy environment even though it is very simple. The AWGN channel model is described by (1) as

$$y = x + z \tag{1}$$

where, x is the channel input, y is the channel output and z is a noise sample drawn from a zero-mean Gaussian distribution with variance σ_z^2 and is assumed to be independent of the channel input. The mutual information between the channel input and output for the Gaussian channel model is given by (2) as

$$I(x;y) = h(y) - h(y|x) = h(y) - h(z) \tag{2}$$

where, $h(\cdot)$ denotes differential entropy of a continuous random variable. Here, $I(x;y)$ is maximized only when y is Gaussian since the normal distribution maximizes the entropy for a given variance. Thus for the output y to be Gaussian, the input x should be Gaussian. The channel capacity is the highest rate in bits per channel symbol at which information can be transferred with low error probability [2]. The AWGN channel capacity, C is given by (3) as

$$C = \frac{1}{2}\log_2(1 + SNR) \quad \text{bits / symbol.} \tag{3}$$

For the sake of simplicity, we have assumed AWGN channel in our study.

III. Superposition Mapping

The general structure of superposition mapping is as depicted in Fig. 1. Here, after Serial-to-Parallel (S/P) conversion, the N input code bits are first converted into binary antipodal symbols d_n via Binary Phase Shift Keying (BPSK). Then, amplitude allocation is done to each of these symbols after which these component symbols are linearly superimposed to create a finite-alphabet output symbol, x [10] . This whole mapping procedure can be mathematically expressed by (4) as

$$x = \sum_{n=1}^{N} c_n = \sum_{n=1}^{N} \alpha_n d_n = \sum_{n=1}^{N} \alpha_n (1 - 2b_n), \tag{4}$$

where $\alpha_n \in \mathbf{R}$, $d_n \in \{\pm 1\}$, $b_n \in \{0,1\}$ and α_n is the magnitude of the n^{th} binary chip. The set of magnitudes $\{\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n\}$ specifies the power allocation among the superimposed chips, c_n. Power allocation significantly influences the supportable bandwidth efficiency and the achievable power efficiency [13]. We have considered one-dimensional signaling in this paper i.e., $\alpha_n \in \mathbf{R}$ for all $n \in \{1, \ldots N\}$.

A. *Equal Power Allocation* (*EPA*)

Equal Power Allocation (EPA) is the simplest yet the most essential power allocation strategy for SM. The chip amplitudes are all identical for EPA and can be expressed as in (5) by

$$\alpha_i = \alpha_j \quad \forall \quad 1 \le i,j \le N \tag{5}$$

Thus a single $\alpha \in \mathbf{R}$ can be used to denote the chip amplitudes. The value of α has been chosen to satisfy $\alpha^2 N = E_s$. In the case of EPA, for power normalization the amplitude coefficient is normally chosen as $\alpha = \sqrt{1/N}$ to ensure that $E\{x^2\} = E_s = 1$ where E_s is the average symbol energy. To obtain simple expressions, $\alpha = 1$ is used for illustration purpose while $\alpha = \sqrt{1/N}$ for the purpose of performance assessment. The cardinality of SM-EPA is defined by $|x| = N + 1$. The parameter x has a Gaussian distribution for large N and the symbol distribution being Gaussian-like, as given below in Fig. 2 for $N = 34$, the necessity of active signal shaping is eliminated. From the definition of entropy, the entropy of SM symbols [13] can be approximated as in (6) by

$$H(x) \approx \frac{1}{2}\log_2\left(\frac{\pi}{2}eN\right) \text{ bits/symbol.} \tag{6}$$

Equation (6) reflects the major drawback SM-EPA in the sense that that the entropy is expected to be $H(x) \sim N$ instead of $H(x) \sim \log_2(N)$. Thus SM-EPA is power efficient in the sense that it delivers a Gaussian-shaped symbol distribution but not bandwidth efficient as the amount of information carried by an SM-EPA symbol is less due to the non-uniform symbol distribution.

Thus, Table I. illustrates the logarithmic variation of symbol entropy of SM-EPA with bit load N and hence the decreasing nature of $H(x)/N$ with increasing N. Here, $H(x)/N$ defines the compression rate of a superposition mapper where N code bits are compressed into an SM symbol carrying $H(x)$bits of information.

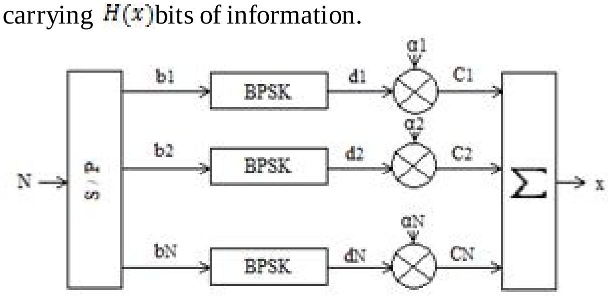

Figure 1. General structure of superposition mapping

B. *Unequal Power Allocation* (*UPA*)

The Unequal Power Allocation scheme is characterized by the exponential law and hence is described by (7) given below as

$$x = \sum_{n=1}^{N} c_n = \sum_{n=1}^{N} \alpha_n d_n \quad , \quad d_n \in \{\pm 1\} \tag{7}$$

with $\alpha_n = a.\rho^{n-1}, 0 < \rho < 1$ where ρ is the exponential base and the value of 'a' should be such that $E\{x^2\} = E_s$ is fulfilled.

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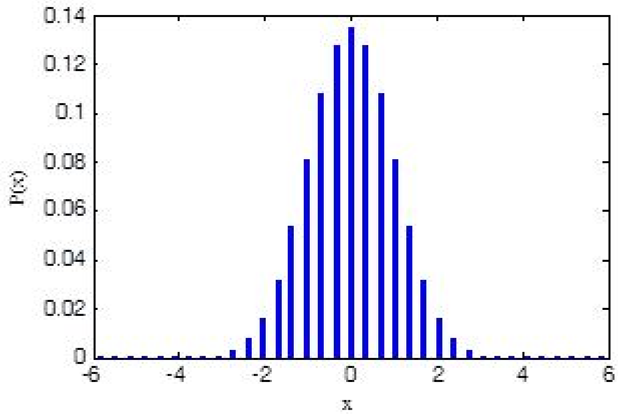


Figure 2. Symbol distribution of SM-EPA, $N = 34$ $E_s = 1, \alpha = \sqrt{1/N}$

TABLE I. SYMBOL CARDINALITIES, ENTROPIES AND COMPRESSION RATES OF SM-EPA

N	$\lvert\mathcal{X}\rvert = N + 1$	$H(x)$	$H(x)/N$
4	5	2.03 bits	0.50
8	9	2.54 bits	0.31
12	13	2.83 bits	0.23
16	17	3.04 bits	0.19

The symbol cardinality of SM-UPA is given by $|\mathcal{X}| = 2^N$. Thus SM-UPA is bijective for $\rho < 1$ [10]. Moreover, the symbol distribution is uniform and probabilistically equal for $\rho = 0.5$ as depicted in Fig. 3 for $N = 6$ but geometrically non-uniform for $\rho \neq 0.5$. The symbol entropy varies linearly with the bit load N and hence is bandwidth efficient. However, due to the non-Gaussian shape of the symbol distribution, SM-UPA is not capacity achieving.

C. Grouped Power Allocation (*GPA*)

Grouped Power Allocation is a hybrid of equal and unequal power allocation strategy. It shows the merits of both EPA and UPA while eliminating the problems from both [10]. Thus SM-GPA is defined in (8) as

$$x = \sum_{l=1}^{L} \alpha_l \sum_{g=1}^{G} d_{l,g} \qquad where\ d_{l,g} \in \{\pm 1\}, \tag{8}$$

where L gives the number of power levels and G gives the group size and $N = L.G$. The amplitude co-efficient α_l of the l^{th} power level is defined as $\alpha_l = \alpha 2^{-(l-1)}$ with the value of 'a' chosen to satisfy $E\{x^2\} = E_s$. SM-GPA symbol cardinality is given by $|\mathcal{X}| = G(2^L - 1) + 1$. The symbol distribution is uniform for $G = 1$ and thus SM-GPA with such a set-up is equivalent to SM-UPA and hence not capacity achieving.

As for $G = 2$, a triangular envelope distribution is obtained as depicted in Fig. 4 (a) for $G = 2$, $L = 3$ which again is not desirable from capacity achieving point of view. Further increasing the group size to three or more groups results in more Gaussian symbol distribution as presented in Fig. 4 (b) for $G = 3$, $L = 3$ which is desirable for achieving Gaussian channel capacity. The group size, G thus solely determines the shape of the symbol distribution and a moderate value of $G = 3$ is sufficient to achieve optimal power efficiency.

Finally with $L = 1$, we have $N = G$, and SM-GPA with such a set-up is equal to SM-EPA. The symbol entropy for SM-GPA [10] for large L can be approximated as

$$H(x) \approx \frac{1}{2} log_2 \left(\frac{\pi}{6} eG . 2^{2L}\right) \sim \frac{1}{2} log_2 \left(\frac{\pi}{6} eG\right) + L$$

bits/symbol. Table II. list the symbol entropies of SM-GPA for different values of group size, G and same power level, L. i.e., for $L = 4$. Thus, SM-GPA is much more efficient than SM-EPA in terms of supportable bandwidth efficiency, given similar bit loads, N which can be illustrated by comparing Table I. and Table II.

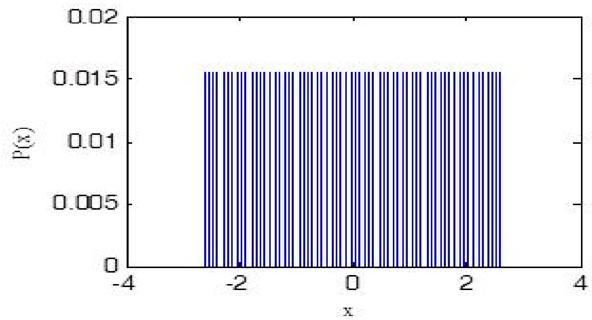
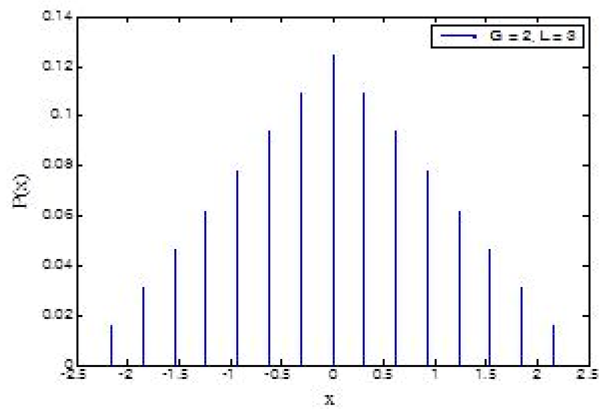
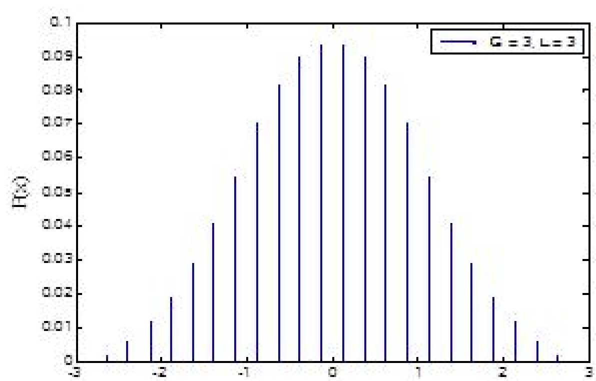


Figure 3. Symbol distribution of SM-UPA, $\rho = 0.5$, $N = 6$



(a) $G = 2$, $L = 3$.



(b) $G = 3$, $L = 3$

Figure 4. Symbol distribution of SM - GPA, $E_s = 1$.

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TABLE II. SYMBOL CARDINALITIES, ENTROPIES AND COMPRESSION RATES OF SM-GPA

G	L	N=GL	$\|\mathcal{X}\|$	$H(x)$ bits/symbol	$H(x)/N$
1	4	4	16	4.25	1.06
2	4	8	31	4.75	0.59
3	4	12	46	5.04	0.42
4	4	16	61	5.25	0.32

IV. SIMULATION RESULTS

Higher modulation techniques such as QAM is still not capacity achieving at high SNRs with active signal shaping. There still exists some gap between the Gaussian capacity curve and the capacity curves of QAM. In the terminology of signal shaping, this gap is often referred to as the ultimate shaping gain which is equal to 1.53 dB, as it gives the maximum possible gain that signal shaping can yield. In this section, the capacity achieving potential of the three power allocation schemes of SM has been compared with that of the conventional mapping technique, QAM. The simulation result illustrates that of the three power allocation schemes SM-GPA proves to be the most effective scheme in achieving the Gaussian channel capacity and also outperforms the conventional mapping technique, QAM in respect of the capacity achieving capability.

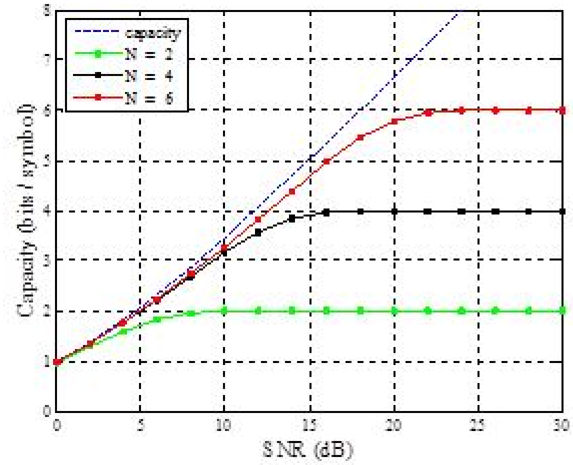
A. SM-EPA

The fact that conventional QAM mapping is not so capacity achieving at high SNRs even with active signal shaping has been demonstrated in Fig. 5 (a). It is observed that some gap still exists between the Gaussian capacity curve and the linear section of the capacity curves for QAM.

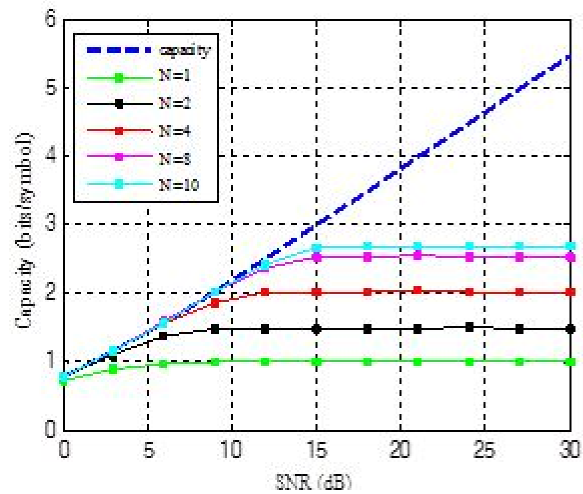
However, for SM-EPA as depicted in Fig. 5 (b) the capacity curves with N = 10 sticks with the Gaussian capacity curve till $SNR \approx 12$ dB which illustrates the capacity achieving potential of SM-EPA at large SNRs as long as the bit load, N is large enough, without active signal shaping as compared to QAM. Also, a larger value of N is required to achieve the same capacity with respect to QAM mapping. As for example, in the figure given, we see that at an SNR of approximately 7 dB a capacity of 2 bits/symbol is achieved with QAM mapping at N = 2 while the same capacity is achieved with SM-EPA at N = 4 and with an SNR of approximately 12 dB which is higher than the SNR value of that of QAM. Thus for SM-EPA at high SNR as the value of N increases, the capacity curves almost sticks with the Gaussian capacity curve thus illustrating the capacity achieving potential of SM-EPA without the necessity of active signal shaping. Thus SM-EPA shows better performance than QAM in capacity achieving potential.

B. SM-UPA

The capacity curves for SM-UPA mapping for ρ = 0. 5 and ρ = 0.30 are demonstrated in Fig. 6 (a) and Fig. 6 (b) respectively. The capacity curves for ρ = 0. 5 are closer towards the Gaussian capacity curve compared with that of ρ = 0.30 where the curves are more deviated from the ideal Gaussian curve. Thus ρ = 0. 5 is a better choice for SM-UPA. Just as with QAM depicted in Fig. 5 (a) SM-UPA is not capacity achieving in the linear section.



(a) QAM



(b) SM-EPA

Figure 5. Capacity vs. SNR curves over AWGN channel

Comparing the capacity curves for QAM given in Fig. 5 (a) with that of SM-UPA with ρ = 0. 5 shown in Fig. 6 (a) we observe that for SM-UPA, a higher value of SNR is required to achieve the same capacity with respect to QAM considering equal values of N. As demonstrated in figure, with QAM a capacity of 6 bits/symbol is achieved at an SNR of approximately 24 dB whereas the same capacity is achieved with SM-UPA with ρ = 0. 5 at an SNR of around 37 dB i.e., at the cost of higher SNR value given the same values of N. Thus SM-UPA does not show enhanced performance w. r. t. QAM.

C. SM-GPA

For the case of G = 1, SM-GPA with such a setup is equivalent to SM-UPA and as the symbol distribution are all

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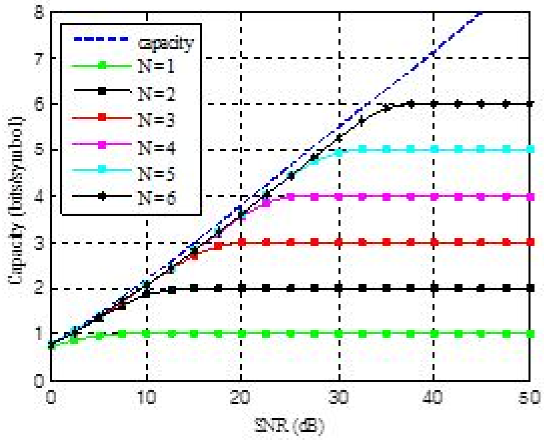
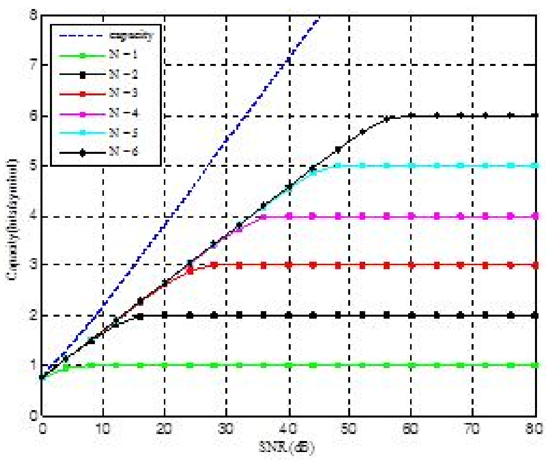
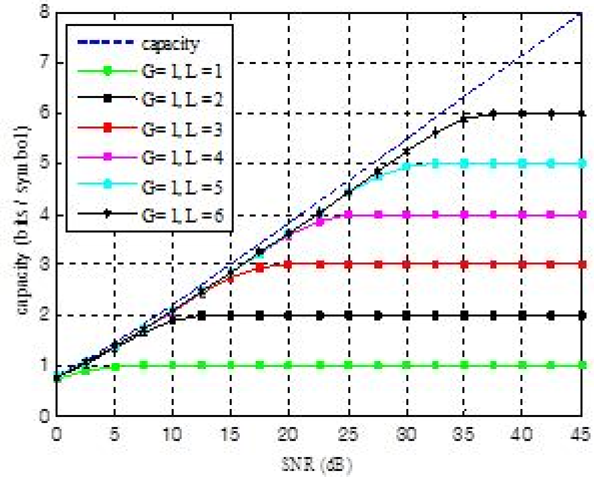


(a) ρ = 0.5

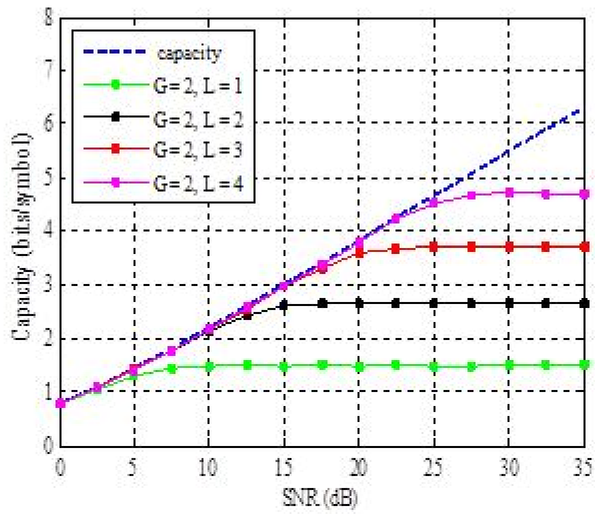


(b) ρ = 0.30

Figure 6. Capacity vs. SNR curves of SM-UPA over AWGN channel



(a) G = 1



(b) G = 2

Figure 7. Capacity vs. SNR curves of SM-GPA over AWGN channel

uniform the respective capacity curves are not capacity achieving in the linear section as in Fig. 7 (a). With $G = 2$, a triangular shaped symbol distribution is obtained as depicted in Fig. 4 (a). A triangular shaped symbol distribution is not optimal but much more better and Gaussian-like than a uniform one. The respective capacity curves depicted in Fig. 7 (b) are almost capacity achieving in the linear section for any value of SNR. This establishes the capacity achieving nature of SM-GPA without active signal shaping. From these simulation results and also from the theoretical results obtained in Table I. and Table II. we can conclude that SM-GPA is both power efficient and bandwidth efficient and hence is the most effective power allocation scheme among the three alternatives in terms of its ability to achieve Gaussian channel capacity and also outperforms QAM in capacity achieving potential without any signal shaping.

IV. Conclusion And Future Work

In this paper, a new technique of non-bijective mapping called Superposition Mapping / Modulation (SM) has been studied with particular focus on its different power allocation schemes. It has been pointed out that SM is Gaussian capacity achieving with proper power allocation and in this respect Grouped Power Allocation is found to be more suitable and achieves Gaussian channel capacity when concerned with real-valued signals. Through proper parameter setting if the symbol distribution of SM-UPA can be made Gaussian-like so as make it power efficient then it could provide excellent performance in terms of the Gaussian capacity achieving capability. This would be a challenging task for the modern researchers. SM shows good performance when applied in Bit-Interleaved Coded Modulation (BICM). The work presented in this paper could further be extended in conjunction with Orthogonal Frequency Division Multiplexing (OFDM) and also in Multi-Input Multi-Output (MIMO) transmission systems.

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